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ON THE CORRECTNESS OF ONE EXTREME PROBLEM RELATED TO INVERSE COEFFICIENT PROBLEMS

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An extremal (variational) problem on the minimum of a certain residual functional is considered. The extremal problem is related to the inverse problem of finding the thermal diffusivity coefficient in the stationary diffusion–advection–reaction model. The residual functional is the difference in some metric between the simulated and observed state of the model. Various aspects of the variational problem are studied. It is shown that the set of minimum points in a variational problem may turn out to be empty. Some conditions for the solvability of the variational problem are also given when the set of minimum points is nonempty. Some conditions for the uniqueness of a minimizing element are indicated. The concepts of weak and strong correctness of an extremal problem are formulated. Examples of problems are given in which both weak and strong correctness is absent, or there is weak correctness but no strong correctness. Some conditions of strong correctness for extreme problem are specified. A necessary minimum condition is formulated in the form of the integral and local maximum principle.

Keywords: diffusion–advection–reaction equation, thermal diffusivity coefficient, inverse problem, residual functional, extremal problem, variational method, minimum point.

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