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FIBONACCI REPRESENTATIONS OF BRAID GROUPS

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A family of representations is constructed for the braid group B_n . The vector spaces on which the braid group acts are defined as the result of identifying the spaces generated by proper colorings of regular trees of degree 3 with a marked vertex. This identification is done using a family of canonical isomorphisms. The dimensions of the resulting spaces form the sequence of Fibonacci numbers. We then show how the constructed representations can be extended to invariants of unoriented knots and links in a 3-sphere.

Keywords: braid group, representation, knot invariant, Reshetikhin–Turaev type invariant.

REFERENCES

1. Bigelow S.L. Braid groups are linear. *J. American Math. Soc.*, 2001, vol. 14, no. 2, pp. 471–486. doi: 10.1090/S0894-0347-00-00361-1
2. Burau W. Über Zopfgruppen und gleichsinnig verdrillte Verkettungen. *Abhandlungen aus dem Mathematischen Seminar der Universität Hamburg*, 1935, vol. 11, pp. 179–186. doi: 10.1007/BF02940722
3. Kassel C., Turaev V. *Braid groups*. Ser. Graduate Texts in Mathematics, vol. 247, 2008, 348 p. doi: 10.1007/978-0-387-68548-9
4. Kauffman L.H., Lomanco S.J. The Fibonacci model and the Temperley–Lieb algebra. *Internat. J. Modern Phys. B*, 2009, vol. 22, no. 29, pp. 5065–5080. doi: 10.1142/S0217979208049303
5. Korablev Ph. *Invariants for links and 3-manifolds from the modular category with two simple objects*. Preprint arXiv:2305.00733 [math.GT]. 28 p. Available at: <https://arxiv.org/abs/2305.00733>. doi: 10.48550/arXiv.2305.00733
6. Krammer D. The braid group B_4 is linear. *Inventiones Mathematicae*, 2000, vol. 142, pp. 451–486. doi: 10.1007/s002220000088
7. Lawrence R.J. Homological representations of the Hecke algebra. *Communic. Math. Phys.*, 1990, vol. 135, no. 1, pp. 141–191. doi: 10.1007/BF02097660
8. Matveev S.V., Ovchinnikov M.A., Sokolov M.V. Construction and properties of the t -invariant. *J. Math. Sci.*, 2003, vol. 113, no. 6, pp. 849–855. doi: 10.1023/A:1021247621259
9. Reshetikhin N., Turaev V. Invariants of 3-manifolds via link polynomials and quantum groups. *Inventiones mathematicae*, 1991, vol. 103, no. 3, pp. 547–597. doi: 10.1007/BF01239527
10. Stanley R.P. *Catalan numbers*. Cambridge: Cambridge University Press, 2015, 215 p. doi: 10.1017/CBO9781139871495
11. Turaev V. Faithful linear representations of the braid groups. *Seminaire Bourbaki*, 1999–2000, vol. 42, pp. 389–409.
12. Turaev V.G. *Quantum invariants of knots and 3-manifolds*. Ser. De Gruyter Studies in Mathematics, 2016, 596 p. doi: 10.1515/9783110435221
13. Turaev V. Operator invariants of tangles, and R -matrices. *Math. USSR Izvestiya*, 1990, vol. 35, pp. 411–444. doi: 10.1070/IM1990v035n02ABEH000711
14. Turaev V., Virelizier A. *Monoidal categories and topological field theory*. Cham, Birkhäuser, 2017, 513 p. doi: 10.1007/978-3-319-49834-8
15. Wang Z. *Topological quantum computation*. CBMS Regional Conference Ser. in Math, 2010, vol. 112, 115 p. doi: 10.1090/cbms/112

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