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On THE EXISTENCE OF A SPORADIC COMPOSITION FACTOR IN SOME FINITE GROUPS

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Assume that G is a finite group, $\pi(G)$ is the set of all prime divisors of its order, and $\omega(G)$ is the set of all orders of its elements (its spectrum). The prime graph (or the Gruenberg–Kegel graph) of a finite group G is a graph $GK(G)$ such that its vertices are the prime divisors of the order of G and two distinct vertices p and q are adjacent in $GK(G)$ if and only if G contains an element of order pq. The prime graphs of nonabelian finite simple groups are known. One of the most popular fields of research in finite group theory is the study of finite groups by the properties of their prime graphs. We study nonabelian composition factors of finite groups whose prime graphs are the same as the prime graphs of known simple groups. In 2011, A.M. Staroletov studied finite groups with a sporadic composition factor whose spectrum is the same as the spectrum of a finite simple group. Generalizing this result, we consider the question of whether a composition factor of a finite group whose prime graph is the same as the prime graph of a finite simple group can be isomorphic to a sporadic group. It is shown that a finite group whose prime graph is the same as the prime graph of a simple exceptional group of Lie type other than $G_2(q)$ and $^3D_4(q)$ or the prime graph of simple classical groups $L_n(q)$, $U_n(q)$, $O_{2n+1}(q)$, and $S_{2n}(q)$ for large enough n has no sporadic composition factors other than F_1. In addition, we describe sporadic composition factors S of finite groups G with the conditions $GK(G) = GK(H)$ and $\pi(G) = \pi(S)$, where H is a simple alternating group or a simple group of Lie type.

Keywords: finite group, simple group, sporadic group, exceptional group of Lie type, classical group, Gruenberg–Kegel graph (prime graph).

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