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ANALYSIS OF NUMERICAL DIFFERENTIATION FORMULAS ON A UNIFORM GRID IN THE PRESENCE OF A BOUNDARY LAYER

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The issue of numerical differentiation of functions with large gradients is considered. It is assumed that there is a decomposition of a given function of one variable into the sum of a regular component and a boundary layer component; the latter is responsible for the large gradients of the function and is known up to a factor. This decomposition is valid, in particular, for a solution of a singularly perturbed boundary value problem. However, the application of the classical polynomial formulas of numerical differentiation to functions with large gradients may produce significant errors. Numerical differentiation formulas that are exact on the boundary layer component are studied, and their error is estimated. Such formulas are proved to be more exact than the classical ones in the case of the presence of a boundary layer component. An approach to estimating the error of the proposed formulas is suggested, and its applicability is shown in particular cases. The results of numerical experiments are presented. These results comply with the obtained error estimates and show the advantage in accuracy of the proposed formulas.

Keywords: function of one variable, large gradients, boundary layer component, nonpolynomial formula for numerical differentiation, error estimation.

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