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ON SOME PROPERTIES OF REACHABLE SETS FOR NONLINEAR SYSTEMS WITH CONTROL CONSTRAINTS IN L_p

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The paper considers reachable sets at a given time for control-affine systems with integral control constraints in the space L_p for $p > 1$. The goal of the paper is to characterize controls leading to the boundary of reachable sets as solutions to extremal problems and to study the necessary optimality conditions in the form of the Pontryagin maximum principle for these controls. A reachable set is interpreted here as the image of the set of admissible controls under a nonlinear mapping defined by a dynamical system. We also study the application of the maximum principle to describe projections of a reachable set onto a subspace and its sections by a hyperplane. The dependence of a reachable set on the control resource is studied. The results obtained are illustrated using the example of linear systems. It is shown that in this case the optimality conditions for boundary controls are necessary and sufficient.

Keywords: control system, integral constraints, reachable set, nonlinear mapping, maximum principle.

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