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THE VALUE AND OPTIMAL STRATEGIES IN A POSITIONAL DIFFERENTIAL GAME FOR A NEUTRAL-TYPE SYSTEM

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On a finite time interval, a differential game for the minimax–maximin of a given cost functional is considered. In this game, the motion of a conflict-controlled dynamical system is described by functional differential equations of neutral type in Hale’s form. Under assumptions more general than those considered previously, a theorem on the existence of the value and saddle point of the game in classes of players’ closed-loop control strategies with memory of the motion history is proved. The proof involves the technique of the corresponding path-dependent Hamilton–Jacobi equations with coinvariant derivatives and the theory of minimax (generalized) solutions of such equations. In order to construct optimal strategies, which constitute a saddle point of the game, a recent result on the existence and uniqueness of a suitable minimax solution and a special Lyapunov–Krasovskii functional are used.

Keywords: differential game, neutral-type equation, game value, optimal strategies, path-dependent Hamilton–Jacobi equation, coinvariant derivatives, minimax solution.

REFERENCES

1. Krasovskii N.N., Subbotin A.I. *Game-theoretical control problems*. NY, Springer, 1988, 517 p. ISBN: 978-1-4612-8318-8. Original Russian text published in Krasovskii N.N., Subbotin A.I. *Pozitsionnye differentsial’nye igrы*, Moscow, Nauka Publ., 1974, 456 p.
2. Krasovskii N.N. *Upravlenie dinamicheskoi sistemoi* [Control of a dynamical system]. Moscow, Nauka Publ., 1985, 520 p.
3. Krasovskii N.N., Krasovskii A.N. *Control under lack of information*. Berlin etc.: Birkhäuser, 1995. 322 p. ISBN: 0-8176-3698-6.
4. Hale J.K., Cruz M.A. Existence, uniqueness and continuous dependence for hereditary systems. *Ann. Mat. Pura Appl.*, 1970, vol. 85, no. 1, pp. 63–81. doi: 10.1007/BF02413530
5. Akhmerov R.R., Kamenskii M.I., Potapov A.S., Rodkina A.E., Sadovskii B.N. Theory of equations of neutral type. *J. Soviet Math.*, 1984, vol. 24, no. 6, pp. 674–719. doi: 10.1007/BF01305757
6. Hale J.K., Lunel S.M.V. *Introduction to functional differential equations*. NY: Springer, 1993, 447 p. doi: 10.1007/978-1-4612-4342-7
7. Lukoyanov N.Yu., Plaksin A.R. Differential games for neutral-type systems: An approximation model. *Proc. Steklov Inst. Math.*, 2015, vol. 291, pp. 190–202. doi: 10.1134/S0081543815080155
8. Gomoyunov M.I., Lukoyanov N.Yu., Plaksin A.R. Existence of a value and a saddle point in positional differential games for neutral-type systems. *Proc. Steklov Inst. Math. (Suppl.)*, 2017, vol. 299, suppl. 1, pp. S37–S48. doi: 10.1134/S0081543817090061
9. Gomoyunov M.I., Plaksin A.R. On basic equation of differential games for neutral-type systems. *Mech. Solids*, 2019, vol. 54, no. 2, pp. 131–143. doi: 10.3103/S0025654419030099
10. Lukoyanov N.Yu., Plaksin A.R. On the theory of positional differential games for neutral-type systems. *Proc. Steklov Inst. Math. (Suppl.)*, 2020, vol. 309, suppl. 1, pp. S83–S92. doi: 10.1134/S0081543820040100
11. Plaksin A.R. Optimal positional strategies in differential games for neutral-type systems. *Dyn. Games Appl.*, 2024. doi: 10.1007/s13235-024-00565-8
12. Garnysheva G.G., Subbotin A.I. Strategies of minimax aiming in the direction of the quasigradient. *J. Appl. Math. Mech.*, 1994, vol. 58, no. 4, pp. 575–581. doi: 10.1016/0021-8928(94)90134-1
13. Subbotin A.I. *Generalized solutions of first order PDEs: The dynamical optimization perspective*. Boston: Birkhäuser, 1995, 314 p. doi: 10.1007/978-1-4612-0847-1

14. Lukoyanov N.Yu. *Funktsional'nye uravneniya Gamil'tona–Yakobi i zadachi upravleniya s nasledstvennoi informatsiei* [Functional Hamilton–Jacobi equations and control problems with hereditary information]. Yekaterinburg: Ural Federal University Publ., 2011, 243 p.
15. Plaksin A.R. On Hamilton–Jacobi–Isaacs–Bellman equation for neutral type systems. *Vestn. Udmurt. Univ. Mat. Mekh. Komp. Nauki*, 2017, vol. 27, no. 2, pp. 222–237 (in Russian). doi: 10.20537/vm170206
16. Gomoyunov M.I., Lukoyanov N.Yu. Minimax solutions of Hamiltoni–Jacobi equations in dynamical optimization problems for hereditary systems. *Uspekhi Mat. Nauk*, 2024, vol. 79, no. 2, pp. 43–144 (in Russian). doi: 10.4213/rm10166
17. Kim A.V. *Functional differential equations. Application of i -smooth calculus*. Dordrecht: Springer, 1999, 168 p. doi: 10.1007/978-94-017-1630-7
18. Plaksin A.R. Minimax solution of functional Hamilton–Jacobi equations for neutral type systems. *Diff. Equat.*, 2019, vol. 55, no. 11, pp. 1475–1484. doi: 10.1134/S0012266119110077
19. Plaksin A.R. On the minimax solution of the Hamilton–Jacobi equations for neutral-type systems: the case of an inhomogeneous Hamiltonian. *Diff. Equat.*, 2021, vol. 57, no. 11, pp. 1516–1526. doi: 10.1134/S0012266121110100
20. Plaksin A.R. Viscosity solutions of Hamilton–Jacobi equations for neutral-type systems. *Appl. Math. Optim.*, 2023, vol. 88, no. 1, art. no 6. 29 p. doi: 10.1007/s00245-023-09980-6
21. Gomoyunov M.I., Lukoyanov N.Yu. Minimax solution of hereditary Hamilton–Jacobi equations for neutral type systems. *Uspekhi Mat. Nauk*, 2024, vol. 79, no. 4, pp. 177–178 (in Russian). doi: 10.4213/rm10182
22. Zhou J. *Viscosity solutions to first order path-dependent HJB equations*. 2020. 25 p. <http://Arxiv.org/abs/2004.02095>. doi: 10.48550/arXiv.2004.02095
23. Zhou J. Viscosity solutions to first order path-dependent Hamilton–Jacobi–Bellman equations in Hilbert space. *Automatica*, 2022, vol. 142, art. no. 110347. 15 p. doi: 10.1016/j.automatica.2022.110347
24. Gomoyunov M.I., Lukoyanov N.Yu., Plaksin A.R. Path-dependent Hamilton–Jacobi equations: the minimax solutions revised. *Appl. Math. Optim.*, 2021, vol. 84, suppl. 1, pp. S1087–S1117. doi: 10.1007/s00245-021-09794-4
25. Filippov A.F. *Differential equations with discontinuous righthand sides*. Dordrecht, Kluwer, 304 p. ISBN: 90-277-2699-X. Original Russian text published in Filippov A.F. *Differentsial'nye uravneniya s razryvnoi pravoi chast'yu*, Moscow, Nauka Publ., 1985, 224 p.

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