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AN OPTIMAL SYNTHESIS FOR A TRIPLE INTEGRATOR WITH A STATE CONSTRAINT

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The time-optimal problem of steering a triple integrator from an arbitrary point to the origin is considered under constraints on the input control and on one of the state variables. An optimal control is synthesized based on the maximum principle in the Dubovitskii–Milyutin form.

Keywords: control system, time optimality, state constraint, maximum principle, switching points, Lebesgue–Stieltjes measure, optimal synthesis.

REFERENCES

1. Pontryagin L.S., Boltyanskii V.G., Gamkrelidze R.V., Mishchenko E.F. *The mathematical theory of optimal processes*. NY, London, Sydney, John Wiley and Sons, Inc., 1962, 360 p. ISBN: 978-0470693810. Original Russian text published in Pontryagin L. S., Boltyanskii V. G., Gamkrelidze R. V., Mishchenko E. F. *Matematicheskaya teoriya optimal'nykh protsessov*, Moscow, Phys. Math. Liter. Publ., 1961, 391 p.
2. Ioffe A.D., Tikhomirov V.M. *Teoriya ekstremal'nykh zadach* [Theory of extremal problems]. Moscow, Nauka Publ., 1974, 480 p. ISBN: 978-5-507-44741-1.
3. Dmitruk A., Samylovskiy I. Optimal synthesis in a time-optimal problem for the double integrator system with a linear state constraint. *J. Dyn. Control Syst.*, 2023, vol. 29, no. 1, pp. 21–42. doi: 10.1007/s10883-021-09589-4
4. Fel'dbaum A.A. On the synthesis of optimal systems with the aid of phase space. *Avtomatika i Telemekhanika*, 1955, vol. 16, no. 2, p. 129–149 (in Russian).
5. Fel'dbaum A.A. *Optimal control systems*. NY: Acad. Press, 1965, 452 p. Original Russian text (2nd ed.) published in Fel'dbaum A.A. *Osnovy teorii optimal'nykh avtomaticheskikh sistem*. Moscow: Nauka Publ., 1966, 623 p.
6. Pavlov A.A. *Sintez releinykh sistem, optimal'nykh po bystrodeistviyu* [Synthesis of relay systems with optimal performance]. Moscow: Nauka Publ., 1966, 390 p.
7. Lee E.B., Markus L. *Foundations of optimal control theory*. NY, London, Sydney: John Wiley & Sons, Inc., 1967, 576 p. Translated under the title *Osnovy teorii optimal'nogo upravleniya*, Moscow, Nauka Publ., 1972, 576 p. ISBN: 0471522635 .
8. Chernous'ko F.L., Shmatkov A.M. Time-optimality synthesis in a third-order system. *Dokl. Akad. Nauk*, 1997, vol. 354, no. 2, pp. 174–177 (in Russian).
9. Akulenko L.D., Kostin G.V. Analytical synthesis of time-optimal control in a third-order system. *J. Appl. Math. Mech.*, vol. 64, no. 4, pp. 509–519. doi: 10.1016/S0021-8928(00)00076-9
10. He Suqin, Hu Chuxiong, Zhu Yu, Tomizuka Masayoshi. Time optimal control of triple integrator with input saturation and full state constraints. *Automatica*, 2020, vol. 122, 109240. doi: 10.1016/j.automatica.2020.109240
11. Dubovitskii A.Ya., Milyutin A.A. Extremum problems in the presence of restrictions. *USSR Comp. Math. Math. Phys.*, 1965, vol. 5, iss. 3, pp. 1–80. doi: 10.1016/0041-5553(65)90148-5
12. Milyutin A.A., Dmitruk A.V., Osmolovsky N.P. *Printsip maksimuma v optimal'nom upravlenii* [Maximum principle in optimal control]. Moscow, Mekh.-Math. Moscow State University, 2004, 73 p.
13. Dmitruk A.V., Osmolovskii N.P. Variations of the v -change of time in problems with state constraints. *Proc. Steklov Inst. Math. (Suppl.)*, 2019, vol. 305, iss. 1, pp. S49–S64. doi: 10.1134/S0081543819040072

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