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# STRONG CONSTRAINTS IN THE DYNAMICS OF SYSTEMS WITH GEOMETRIC SINGULARITIES

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The dynamics of holonomic mechanical systems with geometric singularities of the configuration space, such as branch points, is considered. Classical methods for deriving equations of motion are not applicable in neighborhoods of singular points because there are no generalized coordinates. A new method for analyzing the dynamics of systems with singularities is proposed. Some holonomic (rigid) constraints are replaced by elastic ones (springs). As a result, the singularity disappears, but the number of degrees of freedom of the system increases. With an unlimited increase in spring stiffness, the trajectory of a system with elastic constraints should deviate less and less from the configuration space for the original system with holonomic constraints. A hypothesis has been put forward about the motion of a mechanical system whose configuration space could be represented as a union of two smooth manifolds. The limit transition for the spring stiffness is considered using a specific example. For this purpose, a singular pendulum with a spring is constructed. This two-degree mechanical system can be explicitly parameterized, which simplifies its analytical and numerical modeling. In numerical experiments, the motion of the system is consistent with the hypothesis.

Keywords: constraint realization, constraint reactions, manifolds with singularities, singular point, holonomic constraints, Lagrange multipliers.

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