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**SOME QUESTIONS RELATED TO THE EXTENSION  
OF REACHABILITY PROBLEMS  
IN THE CLASS OF FINITELY ADDITIVE MEASURES**

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Questions related to the extension of reachability problems and aimed at the construction of attraction sets, which are asymptotic analogs of reachable sets in the situation of successive relaxation of the constraint system, are studied. Finitely additive measures with the property of weak absolute continuity with respect to a fixed measure are used as generalized elements; the measure (in the case of control problems) is usually defined as the restriction of the Lebesgue measure to some family of measurable sets. The properties of relaxed reachability problems and the connection of their extensions with attraction sets in the class of usual solutions (controls), as well as the properties of these sets that have the sense of stability when the constraints are relaxed and asymptotic insensitivity when some “part” of the constraints is relaxed, are studied.

Keywords: finitely additive measure, attraction set, weak absolute continuity.

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