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## ON AN ADJOINT TRAJECTORY IN INFINITE-HORIZON CONTROL PROBLEMS

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An optimal control problem is considered on an infinite interval with a weakly overtaking optimality criterion. In such problems, the necessary (D.V. Khlopin, 2023) condition at infinity for such a criterion, compatible with the maximum principle, can give a continuum of solutions of the adjoint system. On the other hand, the Cauchy type formula proposed by A.V. Kryazhinsky and S.M. Aseev (2004) always identifies exactly one adjoint trajectory, which often satisfies the maximum principle within the framework of the problem with a free right end. That is why we find asymptotic assumptions on the system that guarantee the compatibility of the Pontryagin maximum principle and this adjoint trajectory (or its modifications for problems with asymptotic terminal constraints). The asymptotic assumptions obtained in this work develop the recent results by D.V. Khlopin (2018, 2023), S.M. Aseev and V.M. Veliov (2019), and S.M. Aseev (2023).

Keywords: optimal control, Pontryagin's maximum principle, asymptotic endpoint constraint, infinite horizon, uniqueness of the adjoint trajectory, transversality condition at infinity, weakly overtaking optimality.

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