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ON THE CONSTRUCTION OF SOLUTIONS TO A GAME PROBLEM WITH A FIXED END TIME

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We study the game problem of convergence of a conflict-control system with a compact set in a finite-dimensional Euclidean space at a fixed time. The schemes for constructing solutions of the problem are based on the methods of the theory of positional differential games designed by N.N. Krasovskii and A.I. Subbotin in the second half of the 20th century. In general, the problem does not assume that the saddle point condition in the small game is satisfied, and therefore the problem is considered in the minimax formulation. New schemes for the approximate calculation of minimax u -stable paths and bridges are described and justified for a wide class of conflict-control systems. The obtained results constitute a stage of the approximate calculation of solutions to the game problem, which is associated with the discretization of the time interval on which the game takes place.

Keywords: control system, u -stable bridge, u -stable path, solvability set, game problem of convergence.

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