

MSC:34H05, 49N45

DOI: 10.21538/0134-4889-2024-30-3-229-240

**GENERALIZED HOPF FORMULA FOR THE VALUE FUNCTION
IN THE POSITIONAL DIFFERENTIAL GAME “BOY AND CROCODILE”****N. N. Subbotina, A. S. Rodin**

The paper proposes a new formula for the minimax solution to the Cauchy boundary value problem for the Hamilton–Jacobi equation in the case when the Hamiltonian depends on time and the gradient in the phase variable of the minimax solution. This formula is a generalization of the Hopf formula. It is shown using a specific example that this formula is true for the minimax solution of the Hamilton–Jacobi equation in the Cauchy problem, which arises in the positional differential game “Boy and Crocodile.” The proposed formula describes the value function in this differential game.

Keywords: positional differential game, value function, Hamilton–Jacobi equation, Hopf formula, directional derivative, minimax solution.

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Received May 27, 2024

Revised June 6, 2024

Accepted June 24, 2024

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Cite this article as: N. N. Subbotina, A. S. Rodin. Generalized Hopf formula for the value function in the positional differential game “Boy and Crocodile”. *Trudy Instituta Matematiki i Mekhaniki UrO RAN*, 2024, vol. 30, no. 3, pp. 229–240.