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**ON THE ERROR OF CALCULATING THE ATTAINABLE VALUES
OF OBJECTIVE FUNCTIONALS FOR CONTROL SYSTEMS
WITH CONTINUOUS AND DISCRETE TIMES**

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For a wide class of linear systems with aftereffect, the problem of attaining target values by a given system is considered under polyhedral constraints on the control. The aim of the control is set by a finite system of linear functionals ℓ_i , $i = 1, \dots, N$; this is why the more precise term “ ℓ -attainability” is used in the paper. The general form of the functionals makes it possible to consider terminal, multipoint, and integral target conditions and their linear combinations as special cases. For the class of systems under consideration, the problem of ℓ -attainability is reduced to a variant of the moment problem. One of the features of this problem is the account of random disturbances in elements of the moment matrix. These disturbances result in the distortion of the lower and upper (by inclusion) approximations of the ℓ -attainable set. To obtain a guaranteed result, special procedures are proposed, which allow one to build open-loop controls with the following properties. First, the implementation of such controls produces trajectories on which the objective functionals take attainable values. Second, the calculation of attainable values is accompanied by guaranteed estimates of the errors associated with disturbances of elements of the moment matrix. In this case, each coordinate of the vector of target values corresponds not only to an interval of feasible values but also to the corresponding probability density of their distribution. The latter property allows one to give probabilistic characteristics to the errors.

Keywords: control problems, continuous–discrete systems with aftereffect, control with constraints, attainable sets.

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