

MSC: 34A34, 93C20

DOI: 10.21538/0134-4889-2024-30-3-191-206

**EXTREMAL SHIFT IN THE PROBLEM OF TRACKING A DISTURBANCE  
IN A PARABOLIC INCLUSION DESCRIBING  
THE TWO-PHASE STEFAN PROBLEM****V. I. Maksimov, Yu. S. Osipov**

The problem of tracking an unknown nonsmooth in time distributed disturbance of a parabolic inclusion describing the two-phase Stefan problem is studied. The problem is reduced to the problem of open-loop control of some appropriately chosen auxiliary system. The control in this system tracks the unknown disturbance in the mean square, and its construction is based on the results of inaccurate measurements of solutions to the given inclusion and to the auxiliary system. Two algorithms for solving the problem that are stable to noise and calculation errors are presented. The algorithms are based on an appropriate modification of Krasovskii's principle of extremal shift known in the theory of guaranteed control.

Keywords: disturbance tracking, parabolic inclusion.

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Received May 27, 2024

Revised June 7, 2024

Accepted June 10, 2024

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Cite this article as: V.I. Maksimov, Yu.S. Osipov. Extremal shift in the problem of tracking a disturbance in a parabolic inclusion describing the two-phase Stefan problem. *Trudy Instituta Matematiki i Mekhaniki UrO RAN*, 2024, vol. 30, no. 3, pp. 191–206.