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# RECONSTRUCTION OF THE ABSORPTION COEFFICIENT IN A MODEL OF STATIONARY REACTION–CONVECTION–DIFFUSION

**A. I. Korotkii, Yu. V. Starodubtseva**

Direct and inverse problems for the stationary reaction–convection–diffusion model are studied. The direct problem is to find a generalized or strong solution to the corresponding boundary value problems for all given model parameters. Conditions for generalized or strong solvability of the direct problem are given, a priori estimates for solutions are presented, and a continuous dependence of a solution to the direct problem on a number of parameters is established in various metrics. The inverse problem consists of finding the a priori unknown absorption coefficient of a medium, which characterizes the absorption of some substance (or heat sink) in a chemical process. Additional information for solving the inverse problem is the result of measuring the substance (or heat) flow on the accessible part of the boundary of the region where the process takes place. It is proved that the inverse problem is ill-posed. Examples are given showing that the inverse problem is unstable under the disturbance of the measured quantity and may have several solutions. To solve the inverse problem, a variational method based on the minimization of some suitable residual functional (objective functional) is proposed. The extremal properties of the problem of minimizing the residual functional are studied. An explicit analytical formula is found for calculating the gradient of the residual functional, and the corresponding adjoint system and optimality system are written. Several stable iterative methods for minimizing the residual functional are proposed. Numerical modeling of the solution to the inverse problem is carried out.

Keywords: reaction–convection–diffusion equation, direct problem, inverse problem, residual functional, functional gradient, adjoint system, variational method, gradient minimization methods.

## REFERENCES

1. Tikhonov A.N., Arsenin V.Y. *Solutions of Ill-Posed Problems.* Winston and Sons, Washington DC, 1977, 137 p. ISBN:0470991240. Original Russian text published in Tikhonov A.N., Arsenin V.Y. *Metody resheniya nekorrektnyh zadach*, Moscow: Nauka Publ., 1979, 288 p.
2. Ivanov V.K., Vasin V.V, Tanana V.P. *Theory of Linear Ill-Posed Problems and its Applications.* Utrecht: VSP Publ., 2002. 281 p. ISBN 10: 906764367X. Original Russian text published in Ivanov V.K., Vasin V.V, Tanana V.P. *Teoriya lineinyh nekorrektnyh zadach i ih prilozheniya*, Moscow: Nauka Publ., 1978, 206 p.
3. Kabanikhin S.I. *Inverse and Ill-Posed Problems. Theory and Applications.* Walter de Gruyter, 2011 459 p. ISBN: 3110224011, 9783110224016. Original Russian text published in Kabanikhin S.I. *Obratnye i nekorrektnye zadachi*, Novosibirsk: Sibirskoe Nauchnoe Izdatel'stvo, 2009, 457 p.
4. Korotkii A.I., Kovtunov D.A. Reconstruction of boundary regimes in the inverse problem of thermal convection of a high-viscosity fluid. *Proc. Steklov Inst. Math*, 2006, vol. 255, suppl. 2. P. 81–92. doi: 10.1134/S0081543806060071
5. Korotkii A.I., Tsepelev I.A., Ismail-Zadeh A.T. Assimilating data on the free surface of a fluid flow to constrain its viscosity. *Proc. Steklov Inst. Math. (Suppl.)*, 2022, vol. 319, suppl. 1, pp. S162–S174. doi: 10.1134/S0081543822060141
6. Korotkii A.I., Tsepelev I.A. Assimilation of boundary data for reconstructing absorption coefficient in a model of stationary reaction-convection-diffusion. *Proc. Steklov Inst. Math. (Suppl.)*, 2023, vol. 321, suppl. 1, pp. S138–S153. doi: 10.1134/S0081543823030136
7. Chandrasekhar S. *Hydrodynamic and hydromagnetic stability*, Oxford: Clarendon Press, 1961, 652 p. ISBN: 048664071X.
8. Landau L.D., Lifshitz E. *Fluid mechanics.* Oxford: Pergamon Press, 1987, 539 p. ISBN: 9781483161044. Original Russian text published in Landau L.D., Lifshits E.M. *Gidrodinamika,* Moscow: Nauka Publ., 1986, 736 p.

9. Ladyzhenskaya O.A. *The mathematical theory of viscous incompressible flow.* NY: Gordon and Breach, 1987, 224 p. ISBN: 0677207603. Original Russian text published in Ladyzhenskaya O.A. *Matematicheskie voprosy dinamiki vyazkoi neszhimaemoi zhidkosti*, Moscow: Nauka Publ., 1970, 288 p.
10. Alekseev G.V., Tereshko D.A. *Analiz i optimizatsiya v gidrodinamike vyazkoi zhidkosti* [Analysis and optimization in viscous fluid dynamics]. Vladivostok: Dal'nauka Publ., 2008, 365 p. ISBN 5804410458
11. Ladyzhenskaya O.A., Ural'tseva N.N. *Linear and quasilinear elliptic equations.* NY, London: Acad. Press, 1968, 495 p.
12. Ladyzhenskaya O.A. *The boundary value problems of mathematical physics.* Berlin, Heidelberg, NY: Springer-Verlag, 1985, 322 p. doi: 10.1007/978-1-4757-4317-3
13. Mikhailov V.P. *Differentsial'nye uravneniya v chastnykh proizvodnykh* [Partial differential equations]. Moscow: Nauka Publ., 1976, 392 p.
14. Adams R.A. *Sobolev spaces.* NY: Acad. Press, 1975, 268 p.
15. Sobolev S.L. *Some application of functional analysis in mathematical physics.* Providence: Amer. Math. Soc., 1991, 286 p. ISBN 5-02-013756-1
16. Rektorys K. *Variatsionnye metody v matematicheskoi fizike i tekhnike* [Variational methods in mathematics, science and engineering]. Moscow: Mir Publ., 1985, 590 p.
17. Vasil'ev F.P. *Metody optimizatsii.* Moscow: Faktorial Press, 2002, 824 p. ISBN 5-88688-056-9.
18. Nocedal J., Wright S.J. *Numerical Optimization.* NY: Springer, 1999, 664 p.
19. A.I. Korotkii, A.L. Litvinenko. Solvability of a mixed boundary value problem for a stationary reaction–convection–diffusion model. *Trudy Instituta Matematiki i Mekhaniki UrO RAN*, 2018, vol. 24, no. 1, pp. 106–120. doi: 10.21538/0134-4889-2018-24-1-106-120



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*Alexander Illarionovich Korotkii*, Dr. Phys.-Math. Sci., Prof., Krasovskii Institute of Mathematics and Mechanics of the Ural Branch of the Russian Academy of Sciences, Yekaterinburg, 620108 Russia, e-mail: korotkii@imm.uran.ru .

*Yulia Vladimirovna Starodubtseva*, Cand. Sci. (Phys.-Math.), Krasovskii Institute of Mathematics and Mechanics of the Ural Branch of the Russian Academy of Sciences, Yekaterinburg, 620108 Russia, e-mail: starodubtsevayv@yandex.ru .



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