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AN OPTIMAL CONTROL PROBLEM WITH A RELAXED STATE CONSTRAINT

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We explore an optimal control problem in the context of a specified open set representing “undesirable” system states. This problem statement is closely linked to the standard optimal control problem with a state constraint and can be viewed as a relaxation of the latter. The interrelation between these problems is examined. The recently derived necessary first-order optimality conditions for the discussed problem are presented. Additionally, an illustrative example is given.

Keywords: optimal control, differential inclusion, Pontryagin’s maximum principle, refined Euler–Lagrange inclusion, state constraint, discontinuous integrand, risk zone.

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