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ON INTERMEDIATE VALUES OF QUANTIZATION DIMENSIONS OF IDEMPOTENT MEASURES

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The quantization dimension $\dim_{\mathcal{F}}(\xi)$ is defined for any point ξ of spaces of the form $\mathcal{F}(X)$, where $\mathcal{F}$ is a half-normal metrizable functor and X is a metric compact space. An example of a quantization dimension is the classical box dimension $\dim_B$ of closed subsets of a compact space X. In this work, the functor I of idempotent measures or Maslov measures is considered as $\mathcal{F}$. It is known that, for any idempotent measure $\mu \in I(X)$, its (upper and lower) quantization dimensions do not exceed the upper and lower box dimensions, respectively, of the space X. These inequalities motivate the question about intermediate values of the quantization dimensions of idempotent measures. The following theorem is proved: on any metric compact space X of dimension $\dim_B X = a < \infty$, for any numbers $c \in [0, a]$ and $b \in [0, a/2) \cap [0, c]$, there is an idempotent measure whose lower quantization dimension is b and whose upper quantization dimension is c. As follows from this theorem, if a metric compact space X has positive box dimension, then X always has an idempotent measure with a positive lower quantization dimension. Moreover, it is known that a similar statement for the box dimension is not true in the general case, since there exists a metric compact space whose box dimension is 1 such that all of its proper closed subsets are zero-dimensional in the sense of the lower box dimension.

Keywords: idempotent measure, box dimension, quantization dimension, metrizable functor.

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