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### EXISTENCE OF AN OPTIMAL STATIONARY SOLUTION IN THE KPP MODEL UNDER NONLOCAL COMPETITION

We consider a resource distributed on a compact closed connected manifold without edge, for example, on a two-dimensional sphere representing the Earth surface. The dynamics of the resource is governed by a model of the Fisher–Kolmogorov–Petrovsky–Piskunov type with coefficients in the reaction term depending on the total amount of the resource, which makes the model equation nonlocal. Under natural assumptions about the model parameters, it is shown that there is at most one nontrivial nonnegative stationary distribution of the resource. Moreover, in the case of constant distributed resource harvesting, there is a harvesting strategy under which such a distribution maximizes the time-averaged resource harvesting over the stationary states.

Keywords: KPP model, stationary solution, time-averaged harvesting, optimal strategy.

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