TRUDY INSTITUTA MATEMATIKI I MEKHANIKI UrO RAN

Vol. 30 No. 2 **2024**

MSC: 54E35, 47H04, 45G10

DOI: 10.21538/0134-4889-2024-30-2-68-85

THE METHOD OF COMPARISON WITH A MODEL EQUATION IN THE STUDY OF INCLUSIONS IN VECTOR METRIC SPACES

E. S. Zhukovskiy, E. A. Panasenko

For a given multivalued mapping $F : X \rightrightarrows Y$ and a given element $\tilde{y} \in Y$, the existence of a solution $x \in X$ to the inclusion $F(x) \ni \tilde{y}$ and its estimates are studied. The sets X and Y are endowed with vector metrics $\mathcal{P}_X^{E_+}$ and $\mathcal{P}_Y^{M_+}$, whose values belong to cones E_+ and M_+ of a Banach space E and a linear topological space M, respectively. The inclusion is compared with a "model" equation $f(t) = 0$, where $f : E_+ \to M$. It is assumed that f can be written as $f(t) \equiv g(t, t)$, where the mapping $g : E_+ \times E_+ \to M$ orderly covers the set $\{0\} \subset M$ with respect to the first argument and is antitone with respect to the second argument and $-g(0, 0) \in M_+$. It is shown that in this case the equation $f(t) = 0$ has a solution $t^* \in E_+$. Further, conditions on the connection between $f(0)$ and $F(x_0)$ and between the increments of $f(t)$ for $t \in [0, t^*]$ and the increments of $F(x)$ for all x in the ball of radius t^* centered at x_0 for some x_0 are formulated, and it is shown that the inclusion has a solution in the ball under these conditions. The results on the operator inclusion obtained in the paper are applied to studying an integral inclusion.

Keywords: operator inclusion, existence and estimates of solutions, integral inclusion, vector metric space.

REFERENCES

1. Krasovskii N.N. *Stability of motion. Applications of Lyapunov's second method to differential systems and equations with delay*. Stanford, Stanford Univ. Press, 1963, 188 p. Original Russian text published in Krasovskii N.N., *Nekotorye zadachi teorii ustoichivosti dvizheniya*, Moscow, Gos. Izd. Fiz. Mat. Lit., 1959, 211 p.
2. Krasovskii N.N., Subbotin A.I. *Game-theoretical control problems*, NY, Springer, 1988, 517 p. ISBN: 978-1-4612-8318-8 . This book is substantially revised version of the monograph: Krasovskii N.N., Subbotin A.I., *Pozitsionnye differentsial'nye igry*, Moscow, Nauka Publ., 1974, 456 p.
3. Osipov Yu.S., Kryazhimskii A.V. *Inverse problems for ordinary differential equations: dynamical solutions*, London, Gordon and Breach, 1995, 625 p. ISBN: 978-2881249440 .
4. Kryazhimskii A.V., Maksimov V.I. On combination of the processes of reconstruction and guaranteeing control. *Autom. Remote Control*, 2013, vol. 74, no. 8, pp. 1235–1248. doi: 10.1134/S0005117913080018
5. Khlopin D.V., Chentsov A.G. On a control problem with incomplete information: quasi-strategies and control procedures with a model. *Differ. Equ.*, 2005, vol. 41, no. 12, pp. 1727–1742. doi: 10.1007/s10625-006-0009-0
6. Piliya A.D., Fedorov V.I. Singularities of the field of an electromagnetic wave in a cold anisotropic plasma with two-dimensional inhomogeneity. *Soviet Physics — JETP*, 1971, vol. 33, pp. 210.
7. Davydov A.A. Limiting directions singularities of generic implicit higher-order ODEs. *Proc. Steklov Inst. Math.*, 2002, vol. 236, pp. 124–131.
8. Vlasenko L.A., Rutkas A.G. On a differential game in a system described by an implicit differential-operator equation. *Differ. Equ.*, 2015, vol. 51, no. 6, pp. 798–807. doi: 10.1134/S0012266115060117
9. Kantorovich L.V. Some further applications of the Newton method for functional equations. *Vestn. Leningr. Univ.*, 1957, vol. 2, no. 7, pp. 68–103 (in Russian).
10. Kantorovich L.V., Akilov G.P. *Functional Analysis*. Transl. from the 2nd Russian ed. NY, Pergamon Press, 1982, 604 p. doi: 10.1016/C2013-0-03044-7 . Original Russian text (3rd ed.) published in Kantorovich L.V., Akilov G.P. *Funktsional'nyi analiz*, Moscow, Nauka Publ., 1984, 752 p.
11. Zubelevich O. Coincidence points of mappings in Banach spaces. *Fixed Point Theory*, 2020, vol. 21, no. 1, pp. 389–394. doi: 10.24193/fpt-ro.2020.1.27

12. Arutyunov A.V., Zhukovskiy E.S., Zhukovskiy S.E. Kantorovichs fixed point theorem in metric spaces and coincidence points. *Proc. Steklov Inst. Math.*, 2019, vol. 304, suppl. 1, pp. 60–73. doi: 10.1134/S008154381901005X
13. Arutyunov A.V., Zhukovskiy E.S., Zhukovskiy S.E. On the stability of fixed points and coincidence points of mappings in the generalized Kantorovichs theorem. *Topol. Appl.*, 2020, vol. 275, art. no. 107030. doi: 10.1016/j.topol.2019.107030
14. Zhukovskiy E.S. Comparison method for studying equations in metric spaces. *Math. Notes*, 2020, vol. 108, no. 5, pp. 679–687. doi: 10.1134/S0001434620110061
15. Arutyunov A.V., Zhukovskiy E.S., Zhukovskiy S.E., Zhukovskaya Z.T. Kantorovich's fixed point theorem and coincidence point theorems for mappings in vector metric spaces. *Set-Valued Var. Anal.*, 2022, vol. 30, pp. 397–423. doi: 10.1007/s11228-021-00588-y
16. Zhukovskiy E., Panasenko E. Extension of the Kantorovich theorem to equations in vector metric spaces: applications to functional differential equations. *Mathematics*, 2024, vol. 12, no. 1, art. no. 64. doi: 10.3390/math12010064
17. Perov A.I. Multidimensional version of M. A. Krasnoselskii's generalized contraction principle. *Funct. Anal. Appl.*, 2010, vol. 44, no. 1, pp. 69–72. doi: 10.1007/s10688-010-0008-z
18. Zhukovskiy E.S. On coincidence points for vector mappings. *Russian Math.*, 2016, vol. 60, no. 10, pp. 10–22. doi: 10.3103/S1066369X16100030
19. Zhukovskiy E.S. On coincidence points of multivalued vector mappings of metric spaces. *Math. Notes*, 2016, vol. 100, no. 3, pp. 363–379. doi: 10.1134/S0001434616090030
20. Zhukovskii E.S. Perturbations of vectorial coverings and systems of equations in metric spaces. *Sib. Math. J.*, 2016, vol. 57, no. 2, pp. 230–241. doi: 10.1134/S0037446616020063
21. Panasenko E.A. On operator inclusions in spaces with vector-valued metrics. *Proc. Steklov Inst. Math. (Suppl.)*, 2023, vol. 323, suppl. 1, pp. S222–S242. doi: 10.1134/S0081543823060196
22. Fomenko T.N., Yastrebov K.S. The method of searching for zeros of functionals on a conic metric space and its stability issues. *Moscow Univ. Math. Bull.*, 2020, vol. 75, no. 2, pp. 58–64. doi: 10.3103/S0027132220020023
23. Krasnosel'skii M.A., Zabreiko P.P. *Geometrical methods of nonlinear analysis*. Berlin, Springer, 1984, 412 p. ISBN: 978-3-642-69411-0 . Original Russian text published in Krasnosel'skii M.A., Zabreiko P.P., *Geometricheskie metody nelineinogo analiza*, Moscow, Nauka publ., 1975, 511 p.
24. Arutyunov A.V., Zhukovskiy E.S., Zhukovskiy S.E. Coincidence points principle for mappings in partially ordered spaces. *Topol. Appl.*, 2015, vol. 179, pp. 13–33. doi: 10.1016/j.topol.2014.08.013
25. Zhukovskaya T.V., Filippova O.V., Shindyapin A.I. On the extension of Chaplygin's theorem to the differential equations of neutral type. *Russian Universities Reports. Mathematics*, 2019, vol. 24, no. 127, pp. 272–280. doi: 10.20310/2686-9667-2019-24-127-272-280
26. Zhukovskaya T.V., Zhukovskiy E.S., Serova I.D. Some questions of the analysis of mappings of metricand partially ordered spaces. *Vestnik Ross. Univ. Mat.*, 2020, vol. 25, no. 132, pp. 345–358 (in Russian). doi: 10.20310/2686-9667-2020-25-132-345-358
27. Burlakov E.O., Panasenko E.A., Serova I.D., Zhukovskiy E.S. On order covering set-valued mappings and their applications to the investigation of implicit differential inclusions and dynamic models of economic processes. *Advances in Systems Science and Appl.*, 2022, vol. 22, no. 1, pp. 176—191. doi: 10.25728/assa.2022.22.1.1225
28. Serova I.D. Study of the boundary value problem for a differential inclusion. *Vestnik Ross. Univ. Mat.*, 2023, vol. 28, no. 144, pp. 395–405 (in Russian). doi: 10.20310/2686-9667-2023-28-144-395-405
29. Borisovich Yu. G., Gel'man B. D., Myshkis A. D., Obukhovskii V. V. *Vvedenie v teoriyu mnogoznachnykh otobrazhenii i differentsial'nykh vklyuchenii* [Introduction to the theory of multivalued maps and differential inclusions], Moscow, Librokom, 2011, 224 p. ISBN: 978-5-397-01526-4 .

Received February 15, 2024
Revised February 26, 2024
Accepted March 4, 2024

Funding Agency: The work was supported by the Russian Science Foundation (project no. 24-21-00272), https://rscf.ru/project/24-21-00272/ .

Evgeny Semenovich Zhukovskiy, Dr. Phys.-Math. Sci., Prof., Scientific Research Institute of Mathematics, Physics, and Computer Science, Derzhavin Tambov State University, Tambov, 392000

Russia, e-mail: zukovskys@mail.ru .

Elena Aleksandrovna Panasenko, Cand. Phys.-Math. Sci., Docent, Functional Analysis Department, Derzhavin Tambov State University, Tambov, 392000 Russia, e-mail: panlena_t@mail.ru .

Cite this article as: E. S. Zhukovskiy, E. A. Panasenko. The method of comparison with a model equation in the study of inclusions in vector metric spaces. *Trudy Instituta Matematiki i Mekhaniki UrO RAN*, 2024, vol. 30, no. 2, pp. 68–85 .