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**SUFFICIENT OPTIMALITY CONDITIONS FOR HYBRID SYSTEMS  
OF VARIABLE DIMENSION WITH INTERMEDIATE CONSTRAINTS****A. S. Bortakovskii**

An optimal control problem is considered for a hybrid system in which continuous motion alternates with discrete changes (switchings) of the state space and control space. The switching times are determined as a result of minimizing a functional that takes into account the costs of each switching. Sufficient conditions for the optimality of such systems under additional constraints at the switching times are obtained. The application of the optimality conditions is demonstrated using academic examples.

Keywords: hybrid systems, variable dimension, optimal control.

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