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CONTINUOUS DEPENDENCE OF SETS IN A SPACE OF MEASURES AND A PROGRAM MINIMAX PROBLEM

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For conflict-controlled dynamical systems satisfying the conditions of generalized uniqueness and uniform boundedness, the solvability of the minimax problem in the class of generalized controls is studied. The issues of consistency of such an extension are considered; i. e., the possibility of approximating generalized controls in the space of strategic measures by embeddings of ordinary controls is analyzed. For this purpose, the dependence of the set of measures on the general marginal distribution specified on one of the factors of the base space is studied. The continuity of this dependence in the Hausdorff metric defined by the metric corresponding to the $*$-weak topology in the space of measures is established. The density of embeddings of ordinary controls and control-noise pairs in sets of corresponding generalized controls in the $*$-weak topologies is also shown.

Keywords: generalized controls, strategic measures, minimax problem, $*$-weak convergence, Hausdorff metric.

REFERENCES

1. Krasovskii N.N., Subbotin A.I. An alternative for the game problem of convergence. *J. Appl. Math. Mech.*, 1970, vol. 34, no. 6, pp. 948–965. doi: 10.1016/0021-8928(70)90158-9
2. Krasovskii N.N., Subbotin A.I. Game-theoretical control problems. NY, Springer, 1988, 517 p. ISBN: 978-1-4612-8318-8 . Original Russian text published in Krasovskii N.N., Subbotin A.I. *Pozitsionnye differentsial'nye igry*, Moscow, Nauka Publ., 1974, 456 p.
3. Kryazhimskii A.V. On the theory of positional differential games of convergence-evasion. *Sov. Math. Dokl.*, 1978, vol. 19, pp. 408–412.
4. Hopenhayn H. Entry, exit, and firm dynamics in long run equilibrium. *Econometrica*, 1992, vol. 60, no. 5, pp. 1127–1150.
5. Bergin J., Bernhardt D. Anonymous sequential games: Existence and characterization of equilibria. *Economic Theory*, 1995, vol. 5, pp. 461–489. doi: 10.1007/BF01212329
6. Bergin J. On the continuity of correspondence on sets of measures with restricted marginals. *Economic Theory*, 1999, vol. 13, pp. 471–481. doi: 10.1007/s001990050265
7. Bogachev V.I., Popova S.N. Hausdorff distances between couplings and optimal transportation with a parameter. *Math. Sb.*, 2024, vol. 215, no. 1, pp. 33–58 (in Russian). doi: 10.4213/sm9920
8. Warga J. *Optimal control of differential and functional equations*, NY, Acad. Press, 1972, 531 p. doi: 10.1016/C2013-0-11669-8 . Translated to Russian under the title *Optimal'noe upravlenie differentsial'nymi i funktsional'nymi uravneniyami*, Moscow, Nauka Publ., 1977, 624 p.
9. Warga J. Nonsmooth problems with conflicting controls. *SIAM J. Control Optim.*, 1991, vol. 29, pp. 678–701. doi: 10.1137/032903
10. Warga J., Zhu Q.J. A proper relaxation of shifted and delayed controls. *J. Math. Anal. Appl.*, 1992, vol. 169, pp. 546–561. doi: 10.1016/0022-247x(92)90095-u
11. Rosenblueth J.F. Proper relaxation of optimal control problems. *J. Optim. Theory Appl.*, 1992, vol. 74, no. 3, pp. 509–526. doi: 10.1007/BF00940324
12. Serkov D.A., Chentsov A.G. On a property of continuous dependence of sets in the space of measures. In: *Proc. of the 7th Inernet. Conf. on "Nonlinear Analysis and Extremal Problems"* (NLA-2022), Irkutsk, 2022, pp. 106–107. ISBN 978-5-6041814-2-3 .
13. Chentsov A.G. On a single game control problem on minimax. *Izv. AN SSSR, Ser. Tekhn. Kibernetika*, 1975, no. 1, pp. 39–46 (in Russian).

14. Kuratowski K., Mostowski *A. Set theory*. Ser. Studies in Logic and the Foundations of Mathematics, vol. 53, Amsterdam: North-Holland, 1968, 417 p. ISBN: 9780444534170 . Translated to Russian under the title *Teoriya mnozhestv*, Moscow, Mir Publ., 1970, 416 p.
15. Bogachev V.I. *Osnovy teorii mery* [Basics of measure theory]. Vol. 1. Moscow, Izhevsk: Publ. "Regulyar. Khaotich. Dinamik.", 2003, 545 p. ISBN: 5-93972-195-8 .
16. Dunfrod N., Schwartz J. *Linear operators: General theory*. NY, London, Interscience Publ., 1958. ISBN: 9780470226056. Translated to Russian under the title *Lineinye operatory: Obshchaya teoriya*, Moscow, Inostr. Liter. Publ., 1962, 895 p.
17. Chentsov A.G. *Elementy konechno-additivnoi teorii mery, I* [Elements of finitely additive measure theory, I]. Yekaterinburg, Publ. Ural State Tech. Univ. — Ural Polytech. Inst., 2008, 388 p. ISBN: 978-5-321-01408-0 .
18. Billingsley P. *Convergence of probability measures*. NY: Wiley, 1958. ISBN: 9780471072423 . Translated to Russian under the title *Skhodimost' veroyatnostnykh mer*, Moscow, Nauka Publ., 1977, 352 p.
19. Bogachev V.I. *Osnovy teorii mery* [Basics of measure theory]. Vol. 2. Moscow, Izhevsk: NIZ "Regulyar. Khaotich. Dinamik.", 2003, 578 p. ISBN: 5939721966 .
20. Engelking R. *General topology*. Warszawa, PWN, 1977. Translated to Russian under the title *Obshchaya topologiya*, Moscow, Mir Publ., 1986, 752 p.
21. Neveu J. *Bases mathématiques du calcul des probabilités*. Paris, Masson, 1964, 203 p. ISBN: 978-2-225-61787-4. Translated to Russian under the title *Matematicheskie osnovy teorii veroyatnostei*, Moscow, Mir Publ., 1969, 309 p.
22. Dugundji J. An extension of Tietze's theorem. *Pacific J. Math.*, 1951, vol. 1, no. 3, pp. 353–367.
23. Chentsov A.G. Maximin deviation in a differential game. *Diff. Uravn.*, 1976, vol. 12, pp. 848–856 (in Russian).
24. Dieudonné J. *Foundations of modern analysis*. NY: Acad. Press Inc., 1960, 361 p. Translated to Russian under the title *Osnovy sovremennogo analiza*, Moscow: Mir Publ., 1964, 430 p.
25. Constantin A. A uniqueness criterion for ordinary differential equations. *J. Diff. Eq.*, 2023, vol. 342, no. 5, pp. 179–192. doi: 10.1016/j.jde.2022.09.035 .

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