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## BOLZA MINIMIZATION PROBLEMS FOR THE LOTKA–VOLTERRA COMPETITION MODEL

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To study the relationship between the concentrations of healthy and cancer cells in blood cancers, the Lotka–Volterra competition mathematical model is used. Terms containing the control function that specifies the concentration of the drug or the intensity of the therapy that directly kills cancer cells are added to this model. Two types of restrictions imposed on such a control function are considered: lower and upper restrictions and only a lower restriction. The result is the control Lotka–Volterra competition model with two different sets of admissible controls. For such control models, the Bolza problem is to minimize the weighted difference in the concentrations of cancer and healthy cells both at the final time of a given treatment period and throughout this entire period. For the second set of admissible controls, the integral part of the objective function additionally contains a term reflecting the cost of the treatment being performed. The use of the Pontryagin maximum principle allows us to analytically study the features of optimal controls in the considered minimization problems. For the first set of admissible controls, cases are identified and studied in detail when the optimal control is a bang-bang function, as well as cases when, along with bang-bang portions, the control may contain singular regimens. The established results are confirmed by corresponding numerical calculations performed for various parameter values and initial values of the control Lotka–Volterra competition model.

Keywords: Lotka–Volterra competition model, nonlinear control system, Bolza minimization problem, Pontryagin maximum principle, switching function, bang-bang control, singular regimen, indicator function.

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