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INTEGRO-DIFFERENTIAL EQUATIONS OF GERASIMOV TYPE WITH SECTORIAL OPERATORS

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The issues of existence and uniqueness of a solution to the Cauchy problem are studied for a linear equation in a Banach space with a closed operator at the unknown function that is resolved with respect to a first-order integro-differential operator of the Gerasimov type. The properties of resolving families of operators of the homogeneous equations are investigated. It is shown that sectoriality, i.e., belonging to the class of operators \mathcal{A}_K introduced here, is a necessary and sufficient condition for the existence of an analytical resolving family of operators in a sector. A theorem on the perturbation of operators of the class \mathcal{A}_K is obtained, and two versions of the theorem on the existence and uniqueness of a solution to a linear inhomogeneous equation are proved. Abstract results are used to study initial–boundary value problems for an equation with the Prabhakar time derivative and for a system of partial differential equations with Gerasimov–Caputo time derivatives of different orders.

Keywords: integro-differential equation, Gerasimov–Caputo derivative, Cauchy problem, sectorial operator, resolving family of operators, initial–boundary value problem.

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