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## ON THE PROBLEM OF OPTIMAL STIMULATION OF DEMAND

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We study the problem of optimal stimulation of demand based on a controlled version of Kaldor's business cycle model. Using the approximation method, we prove a version of Pontryagin's maximum principle in the normal form, containing an additional pointwise condition on the adjoint variable. The results obtained develop and strengthen the previous results in this direction.

Keywords: optimal control, Kaldor's business cycle model, Pontryagin's maximum principle.

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