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PACKAGE GUIDANCE PROBLEM FOR A FRACTIONAL-ORDER SYSTEM

P. G. Surkov

The problem of guaranteed closed-loop guidance to a given set at a given time is studied for a linear dynamical control system described by differential equations with a fractional derivative of the Caputo type. The initial state is a priori unknown, but belongs to a given finite set. The information on the position of the system is received online in the form of an observation signal. The solvability of the guidance problem for the control system is analyzed using the method of Osipov–Kryazhinskii program packages. The paper provides a brief overview of the results that develop the method of program packages and use it in guidance problems for various classes of systems. This method allows us to connect the solvability condition of the guaranteed closed-loop guidance problem for an original system with the solvability condition of the open-loop guidance problem for a special extended system. Following the technique of the method of program packages, a criterion for the solvability of the considered guidance problem is derived for a fractional-order system. In the case where the problem is solvable, a special procedure for constructing a guiding program package is given. The developed technique for analyzing the guaranteed closed-loop guidance problem and constructing a guiding control for an unknown initial state is illustrated by the example of a specific linear mechanical control system with a Caputo fractional derivative.

Keywords: control, incomplete information, linear systems, Caputo fractional derivative.

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Platon Gennad'evich Surkov, Cand. Sci. (Phys.-Math.), Krasovskii Institute of Mathematics and Mechanics of the Ural Branch of the Russian Academy of Sciences, Yekaterinburg, 620108 Russia; Ural Federal University, Yekaterinburg, 620000 Russia, e-mail: spg@imm.uran.ru .

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