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ON A CONTROL RECONSTRUCTION PROBLEM WITH NONCONVEX CONSTRAINTS

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A control reconstruction problem for dynamic deterministic affine-controlled systems is considered. This problem consists of constructing piecewise constant approximations of an unknown control generating an observed trajectory from discrete inaccurate measurements of this trajectory. It is assumed that the controls are constrained by known nonconvex geometric constraints. In this case, sliding modes may appear. To describe the impact of sliding modes on the dynamics of the system, the theory of generalized controls is used. The notion of normal control is introduced. It is a control that generates an observed trajectory and is defined in a unique way. The aim of reconstruction is to find piecewise constant approximations of the normal control that satisfy given nonconvex geometric constraints. The convergence of approximations is understood in the sense of weak convergence in the L^2 space. A solution to the control reconstruction problem is proposed.

Keywords: inverse problems, control reconstruction, sliding modes, nonconvex constraints, weak convergence, generalized controls.

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