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**CONTROL OF ACCELERATION OF A DYNAMIC OBJECT
BY THE MODIFIED LINEAR TANGENT LAW
IN THE PRESENCE OF A STATE CONSTRAINT**

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The paper is devoted to trajectory optimization for an inertial object moving in a plane with thrust bounded in absolute value in the presence of external forces. The aim is to maximize the longitudinal terminal velocity with the state constraint satisfied at each time to avoid a lateral collision with an obstacle. The linear tangent law is used as the basis for an algorithm that controls the direction of the thrust. Conditions for the existence of a solution are studied. Constraints on the initial lateral velocity and the time of the motion of the object are obtained. Since the linear tangent law violates the constraint for some motion times, a modified control law is proposed. A transcendental equation is obtained to find a critical value of time above which an undesired collision occurs. The corresponding conjecture is formulated, which allows us to eliminate the ambiguity that arises during the solution process. A method for solving the problem is presented and confirmed by numerical calculations.

Keywords: trajectory optimization, state constraint, velocity maximization, linear tangent law.

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