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ON THE CONTINUITY OF THE OPTIMAL TIME AS A FUNCTION OF THE INITIAL STATE FOR LINEAR CONTROLLED OBJECTS WITH INTEGRAL CONSTRAINTS ON CONTROLS

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A traditional object of study in the mathematical theory of optimal control is a controlled object with geometric constraints on the control vector u. At the same time, it turns out that sometimes it is more convenient to impose integral constraints on the control vector u. For example, in the theory of automatic design of optimal controllers, it is sometimes assumed that the control vector u is not subject to any geometric constraints, but there is a requirement that the control $u(t)$ and its squared length $|u(t)|^2$ are Lebesgue summable on the corresponding interval. This circumstance, as well as the fact that the quality criterion has the form of a quadratic functional, makes it possible to construct an optimal control under rather broad assumptions. Quadratic integral constraints on controls can be interpreted as some energy constraints. Controlled objects under integral constraints on the controls are given quite a lot of attention in the mathematical literature on control theory. We mention the works of N.N. Krasovskii, E.B. Lee, L. Markus, A.B. Kurzhanski, M.I. Gusev, I.V. Zykov, and their students. The paper studies a linear time-optimal problem, in which the terminal set is the origin, under an integral constraint on the control. Sufficient conditions are obtained under which the optimal time as a function of the initial state x_0 is continuous.

Keywords: control, controlled object, integral constraint, time optimality.

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