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## ON SOLVABILITY OF THE TRACKING PROBLEM IN NONLINEAR VECTOR OPTIMIZATION OF OSCILLATION PROCESSES

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The tracking problem is investigated in the nonlinear vector optimization of oscillation processes described by integro-differential partial differential equations when the scalar function of external and boundary influence depends nonlinearly on several controls. It is established that this problem has some specific features; in particular, the components of the distributed and boundary vector controls satisfy a system of equal relations and are defined as a solution to a system of two nonlinear integral equations. A method for solving this system is developed. Sufficient conditions are found for the unique solvability of the tracking problem, and an algorithm is developed for constructing a complete solution to the nonlinear optimization problem.

Keywords: tracking problem, nonlinear optimization, maximum principle, properties of equal ratios, distributed vector optimal control, boundary vector optimal control, optimal process, minimum value of the functional.

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