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A STUDY OF REGULARIZATION FOR A DEGENERATE PROBLEM OF IMPULSIVE STABILIZATION IN A SYSTEM WITH AFTEREFFECT

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Yu. F. Dolgii, A. N. Seseikin. A study of regularization for a degenerate problem of impulsive stabilization in a system with aftereffect.

A degenerate problem of stabilization of a linear autonomous system of differential equations with aftereffect and impulse controls is considered. For its regularization, a non-degenerate criterion for the quality of transient processes is used, which is close to a degenerate one. The regularized stabilization problem for impulse controls is replaced by an auxiliary non-degenerate optimal stabilization problem for non-impulse controls containing aftereffect. Bellman's dynamic programming principle is used to solve the auxiliary problem. When finding the governing system of equations for the coefficients of the quadratic Bellman functional, the formulation of the optimal stabilization problem in the functional spaces of states and controls is used. A representation is obtained for the pulse of the optimal stabilizing control. The difficult problem of finding a solution to the governing system of equations for the Bellman functional is replaced by the problem of finding a solution to the governing system of equations for the coefficients of the representation of the optimal stabilizing control. The latter problem has lower dimension. The asymptotic dependence of the optimal stabilizing control on the regularization parameter is found when the dimension of the control vector coincides with the dimension of the state vector.

Keywords: linear autonomous system, aftereffect, optimal stabilization, impulse control.

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