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**NORMALIZERS OF SYLOW SUBGROUPS IN SYMPLECTIC AND ORTHOGONAL GROUPS OVER FINITE FIELDS OF ODD CHARACTERISTIC****A. S. Vasilev**

The paper identifies the normalizers of Sylow  $r$ -subgroups for an odd prime  $r$  in symplectic and orthogonal groups (both simple and complete) over fields of odd characteristic different from  $r$ . The motivation for this study comes from the fundamental role of  $r$ -subgroups and their normalizers ( $r$ -local subgroups) in the theory of finite groups and the incomplete description of Sylow subgroup normalizers in simple groups as of today. The findings of the work bring us closer to a full description of the normalizers of Sylow  $r$ -subgroups in classical groups. The only case that remains open is for odd  $r$  in symplectic and orthogonal groups over a field of characteristic 2.

Keywords: symplectic groups, orthogonal groups, normalizers of Sylow subgroups, finite simple groups.

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