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## ON SOME COMPLEMENTS TO LIU'S THEORY

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In the framework of Baoding Liu's uncertainty theory, some new concepts are introduced and their properties are considered. In particular, regular functions of uncertainty are introduced on an uncountable product of spaces. An analog of the Lomnitskii–Ulam theorem from traditional probability theory is obtained. Necessary and sufficient conditions are specified under which a function defined on a Banach space of bounded functions is a distribution function for some uncertain mapping. Some notions of Liu's theory are generalized for uncountably many objects. Examples showing the similarity and the difference between Liu's theory and probability theory are analyzed. An application of Liu's theory to estimation theory is considered on examples.

Keywords: functions of uncertainty, uncertain mappings, distribution functions, theory of estimation.

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