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COLLOCATION METHODS WITH FOURTH DEGREE POLYNOMIALS ON TRIANGULAR GRIDS AND THEIR APPLICATION TO THE CALCULATION OF BENDING OF ROUND PLATES WITH HOLES

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A new collocation method (h-CM$_4$) is developed for the numerical solution of two-dimensional elliptic problems with second-order highest derivatives. Fourth-degree polynomials on triangular cells of a grid generated by Gmsh are used as an approximation. Unknown coefficients of the polynomial decomposition are determined from the solution of a system of linear algebraic equations (SLAE) consisting of collocation equations, matching conditions, and boundary conditions. In the h-CM$_4$, the SLAE is quadratic in contrast to published versions of the least-squares collocation method, where similar equations are written, but the SLAE is overdetermined. This leads to an increase in computation time and the need to search for special values of the weight coefficients multiplying the equations of the approximate problem. The fourth order of convergence of the h-CM$_4$ is established numerically on smooth test solutions of the Poisson's equation and of a system of partial differential equations (PDEs) arising in the calculation of bending within the Reissner–Mindlin plate theory (RMPT). The possibility of calculation of the stress–strain state (SSS) of sufficiently thin plates in the RMPT is demonstrated. It is shown that in order to solve the PDE system describing the plate bending within the Kirchhoff–Love plate theory (KLPT) in a mixed formulation, it is necessary to increase the number of equations of the approximate problem in the h-CM$_4$. Thus, the approximation is reduced to the construction of a new version of the least-squares collocation method (h-LSCM$_4$), whose convergence order is no worse than the third. The SSS of round plates with holes is analyzed depending on the thickness of a plate in the RMPT and KLPT as well as on eccentricity in the case of one hole. Adaptive grids are used to improve accuracy in problems with large gradients and limited smoothness of the solution, which resulted in improving the order of convergence in the latter case. The application of adaptive grids expands the capabilities of the h-CM$_4$ and h-LSCM$_4$ compared to previous versions of the least-squares collocation method, which is confirmed by numerical examples.

Keywords: collocation method, Poisson's equation, Reissner–Mindlin theory, Kirchhoff–Love theory, plate bending.

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