

MSC: 05A17, 05C07, 05C35

DOI: 10.21538/0134-4889-2024-30-1-32-42

## ON LATTICES ASSOCIATED WITH MAXIMAL GRAPHICAL PARTITIONS

V. A. Baransky, V. V. Zuev

The aim of this paper is to describe, for a given graphical partition  $\lambda$  of weight  $2m$  and rank  $r$ , the set of all maximal graphical partitions  $\mu$  of weight  $2m$  that dominate  $\lambda$ . To do this, it is enough to find the set of heads of such partitions. Theorem 1 states that, for any natural number  $t$ , the set of heads of all maximal graphical partitions  $\mu$  of weight  $2m$  and rank  $t$  dominating  $\lambda$  forms an interval of the integer partition lattice if such partitions  $\mu$  of rank  $t$  exist. Algorithms are also provided for finding the smallest and largest elements of this interval.

Keywords: lattice, integer partition, Ferrers diagram, graph, maximal graphical partition.

### REFERENCES

1. Andrews G.E. *The theory of partitions*. Cambridge: Cambridge University Press, 1976. 255 p.
2. Brylawski T. The lattice of integer partitions. *Discrete Math.*, 1973, vol. 6, no. 3, pp. 201–219. doi: 10.1016/0012-365X(73)90094-0
3. Baransky V.A., Senchonok T.A. On maximal graphical partitions that are the nearest to a given graphical partition. *Sib. Elect. Math. Reports*, 2020, vol. 17, pp. 338–363 (in Russian). doi: 10.33048/semi.2020.17.022
4. Erdős P., Gallai T. Graphs with given degree of vertices. *Math. Lapok*, 1960, vol. 11, pp. 264–274.
5. Sierksma G., Hoogeveen H. Seven criteria for integer sequences being graphic. *J. Graph Theory*, 1991, vol. 15, no. 2. P. 223–231. doi: 10.1002/jgt.3190150209
6. Kohnert A. Dominance order and graphical partitions. *Elec. J. Comb.*, 2004, vol. 11, no. 1, art. no. N4, pp. 1–17. doi: 10.37236/1845
7. Baransky V.A., Senchonok T.A. On maximal graphical partitions. *Sib. Electron. Mat. Izv.*, 2017, vol. 14, pp. 112–124 (in Russian). doi: 10.17377/semi.2017.14.012
8. Mahadev N.V.R., Peled U.N. *Threshold graphs and related topics*. Ser. Annals of Discr. Math., vol. 56, Amsterdam: North-Holland Publishing Co., 1995, 542 p.

Received November 30, 2023

Revised December 19, 2023

Accepted December 25, 2023

Vitaly Anatolievich Baransky, Dr. Phys.-Math. Sci., Prof., Ural Federal University Yekaterinburg, 620000 Russia, e-mail: vitaly.baransky@urfu.ru .

Valentin Viktorovich Zuev, doctoral student, Ural Federal University Yekaterinburg, 620000 Russia, e-mail: valentin.zuev@urfu.ru .

Cite this article as: V. A. Baransky, V. V. Zuev. On lattices associated with maximal graphical partitions. *Trudy Instituta Matematiki i Mekhaniki UrO RAN*, 2024, vol. 30, no. 1, pp. 32–42.