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FINITE GROUPS WHOSE COMMUTING GRAPH IS SPLIT

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As a contribution to the study of graphs defined on groups, we show that for a finite group G the following statements are equivalent: the commuting graph of G is a split graph; the commuting graph of G is a threshold graph; either G is abelian, or G is a generalized dihedral group $D(A) = \langle A, t : (\forall a \in A)(at)^2 = 1 \rangle$ where A is an abelian group of odd order.

Keywords: commuting graph, split graph, threshold graph, generalized dihedral group.

REFERENCES

1. Bhowal P., Cameron P.J., Nath R.K., and Sambale B. Solvable conjugacy class graph of groups. *Discrete Math.*, 2023, vol. 346, no. 8, art. no. 113467. doi: 10.1016/j.disc.2023.113467
2. Brauer R. and Fowler K.A. On groups of even order. *Ann. Math. (2)*, 1955, vol. 62, pp. 565–583.
3. Britnell J.R. and Gill N. Perfect commuting graphs. *J. Group Theory*, 2017, vol. 20, pp. 71–102. doi: 10.1515/jgth-2016-0024
4. Cameron P.J. Graphs defined on groups. *Internat. J. Group Theory*, 2022, vol. 11, pp. 53–107. doi: 10.22108/IJGT.2021.127679.1681
5. Cameron P.J., Manna P., and Mehatari R., On finite groups whose power graph is a cograph. *J. Algebra*, 2022, vol. 591, pp. 59–74. doi: 10.1016/j.jalgebra.2021.09.034
6. Chvátal V. and Hammer P.L. Aggregations of inequalities in integer programming. *Ann. Discrete Math.*, 1977, vol. 1, pp. 145–162. doi: 10.1016/S0167-5060(08)70731-3
7. Foldes S. and Hammer P.L. Split graphs. In: Proceedings of the 8th South-Eastern Conference on Combinatorics, Graph Theory and Computing, 1977, pp. 311–315.
8. Landau E. Über die Klassenzahl der binären quadratischen Formen von negativer Discriminante. *Math. Ann.*, 1903, vol. 56, pp. 671–676.

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