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## AN OPTIMAL INTERPOLATION PROBLEM WITH HERMITE INFORMATION IN THE SOBOLEV CLASS $W_1^n([-1, 1])$ <sup>1</sup>

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In this paper, we study the optimal interpolation problem in the Sobolev class  $W_1^n([-1, 1])$ ,  $n \in \mathbb{N}$ , with Hermite information. By some properties of spline functions, we proved that the Lagrange interpolation based on the extreme points of Chebyshev polynomials is optimal for  $W_1^n([-1, 1])$ , and we obtained the approximation error for the optimal interpolation problem.

Keywords: Hermite interpolation, spline function, optimal interpolation, Sobolev class.

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