

MSC: 20E25

DOI: 10.21538/0134-4889-2024-30-1-213-222

## ON GROUPS WITH FROBENIUS–ENGEL ELEMENTS

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A number of properties of periodic and mixed groups with Frobenius–Engel elements are found (Lemmas in Sect. 2 and Theorem 1). The results obtained are used to describe mixed and periodic groups with finite elements saturated with finite Frobenius groups. It is proved that a binary finite group saturated with finite Frobenius groups is a Frobenius group with locally finite complement (Theorem 2). Theorem 3 establishes that in a saturated Frobenius group of a primitive binary finite group  $G$  without involutions the characteristic subgroup  $\Omega_1(G)$  generated by all elements of prime orders from  $G$  is a periodic Frobenius group with kernel  $F$  and locally cyclic complement  $H$ . Moreover, any maximal periodic subgroup  $T$  of  $G$  is a Frobenius group with kernel  $F$  and complement  $T \cap N_G(H)$ . A number of examples of periodic non-locally finite and mixed groups satisfying Theorem 3 are given.

Keywords: Frobenius groups, finite elements, Engel elements, Frobenius elements, Frobenius–Engel elements, saturation.

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doi: 10.1007/BF01091650

Received October 18, 2023

Revised February 1, 2024

Accepted February 5, 2024

**Funding Agency:** This work was supported by the Russian Science Foundation (project no. 19-71-10017).

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Cite this article as: A. I. Sozutov. On groups with Frobenius–Engel elements. *Trudy Instituta Matematiki i Mekhaniki UrO RAN*, 2024, vol. 30, no. 1, pp. 213–222.