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ON THE PRODUCT OF OPERATOR FUNCTIONS

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In a Banach space, a linear densely defined operator A and some closed domain \overline{G} lying in the regular set of A and containing the nonpositive real semiaxis are given. A power estimate for the norm of the resolvent of A in the domain G is assumed to be known. Under the assumption that the operators e^{uA} defined by power operator series are closed for $u > 0$, two classes of functions of this operator are introduced and studied. The construction of these classes is based on the integral Cauchy formula with corresponding scalar functions analytic in the complement of G and such that their modules have an exponential estimate in the complement of G . If the operator A satisfies certain constraints, then the introduced classes of functions of A are extensions of the corresponding classes of operator functions, which we studied earlier jointly with L. F. Korkina. The multiplicative property of the operator functions is established, and the question of their invertibility is considered.

Keywords: functions of an operator, operator exponent, multiplicative property.

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