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ON THE SHEAF REPRESENTATION OF A PQ -BAER $*$ -SEMIRING WITH INVOLUTION

N. S. Protasov, V. V. Chermnykh

В статье получено функциональное (пучковое) представление pq -бэровского полукольца с инволюцией ($*$ -полукольца). Для $*$ -полукольца вводятся понятия центрального и центрального первичного идеалов. Множество $\text{Sp } S$ всех центральных первичных идеалов pq -бэровского $*$ -полукольца с топологией Зарисского становится нульмерным компактным хаусдорфовым пространством. На $\text{Sp } S$ как на базисном пространстве строится пучок $*$ -полуколец $(\mathbb{L}(S), \text{Sp } S)$. Доказано, что произвольное pq -бэровское $*$ -полукольцо $*$ -изоморфно $*$ -полукольцу всех глобальных сечений пучка $\mathbb{L}(S)$. Сформулированы открытые вопросы.

Ключевые слова: полукольцо с инволюцией, pq -бэровское $*$ -полукольцо, пучковое представление.

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In the paper, a functional (sheaf) representation of a pq -Baer $*$ -semiring with involution is obtained. For a $*$ -semiring, the notions of central and central prime ideals are introduced. The set $\text{Sp } S$ of all central prime ideals of a pq -Baer $*$ -semiring with the Zariski topology becomes a zero-dimensional compact Hausdorff space. The sheaf $(\mathbb{L}(S), \text{Sp } S)$ of $*$ -semirings is constructed on $\text{Sp } S$ as a basis space. It is proved that an arbitrary pq -Baer $*$ -semiring is $*$ -isomorphic to the $*$ -semiring of all global sections of the sheaf $\mathbb{L}(S)$. Open questions are formulated.

Keywords: semiring with involution, pq -Baer $*$ -semiring, sheaf representation.

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Nikita Sergeevich Protasov, doctoral student, Pitirim Sorokin Syktyvkar State University, Syktyvkar, 167001 Russia, e-mail: protasovnekit@gmail.com .

Vasily Vladimirovich Chermnykh, Dr. Phys.-Math. Sci., Prof., Pitirim Sorokin Syktyvkar State University, Syktyvkar, 167001 Russia, e-mail: vv146@mail.ru .

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