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REIDEMEISTER TORSION FOR VECTOR BUNDLES ON $\mathbb{P}^1_{\mathbb{Z}}$

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We consider vector bundles of rank 2 with a trivial generic fiber on the projective line over $\mathbb{Z}$. For such bundles, a new invariant is constructed — the Reidemeister torsion, which is an analog of the classical Reidemeister torsion from topology. For vector bundles of rank 2 with a trivial generic fiber and jumps of height 1, that is, for the bundles that are isomorphic to $\mathcal{O}^2$ in the fiber over $\mathbb{Q}$ and are isomorphic to $\mathcal{O}^2$ or $\mathcal{O}(-1)\oplus\mathcal{O}(1)$ over each closed point Spec($\mathbb{Z}$), we calculate this invariant and show that it, together with the discriminant of the bundle, completely determines such a bundle.

Keywords: vector bundle, arithmetic surface, projective line, torsion.

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