

MSC: 20D20, 20D35

DOI: 10.21538/0134-4889-2024-30-1-100-108

## FINITE GROUPS WITH $\mathbb{P}$ -SUBNORMAL SCHMIDT SUBGROUPS

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A subgroup  $H$  of a group  $G$  is called  $\mathbb{P}$ -subnormal in  $G$  whenever either  $H = G$  or there is a chain of subgroups

$$H = H_0 \subset H_1 \subset \dots \subset H_n = G$$

such that  $|H_i : H_{i-1}|$  is a prime for every  $i = 1, 2, \dots, n$ . We study the structure of a finite group  $G$  all of whose Schmidt subgroups are  $\mathbb{P}$ -subnormal. The obtained results complement the answer to Problem 18.30 in the *Kourovka Notebook*.

Keywords: finite group,  $\mathbb{P}$ -subnormal subgroup, Schmidt subgroup, saturated Fitting formation.

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Received December 5, 2023

Revised January 8, 2024

Accepted January 15, 2024

**Funding Agency:** The research of the first author was supported by the National Natural Science Foundation of China (grant no. 12371021). The research of the third author was supported by the Belarusian Republican Foundation for Fundamental Research (project no.  $\Phi 23PH\Phi$ -237).

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Cite this article as: X. Yi, Zh. Xu, S. F. Kamornikov. Finite groups with  $\mathbb{P}$ -subnormal Schmidt subgroups. *Trudy Instituta Matematiki i Mekhaniki UrO RAN*, 2024, vol. 30, no. 1, pp. 100–108.