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On finite groups isospectral to $PSp_4(q)$

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The spectrum of a finite group is the set of its element orders. Let q be a power of a prime p , with $p \geq 5$. It is known that any finite group having the same spectrum as the simple symplectic group $PSp_4(q)$ either is isomorphic to an almost simple group with socle $PSp_4(q)$ or can be homomorphically mapped onto an almost simple group H with socle $PSL_2(q^2)$. We prove that the group H cannot coincide with $PSL_2(q^2)$, i.e., H must contain outer automorphisms of its socle.

Keywords: finite group, element order.

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