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ON THE BEST SIMULTANEOUS APPROXIMATION OF FUNCTIONS IN THE HARDY SPACE M. Sh. Shabozov

In the Hardy spaces $H_{q,\rho}$ ($1 \leq q \leq \infty$, $0 < \rho \leq 1$), exact inequalities are found between the best simultaneous approximation of a function and the averaged moduli of smoothness of the angular boundary values of the r th derivatives. Some applications of these inequalities to the problem of finding the best upper bounds of the best simultaneous approximations of some classes of functions defined by moduli of smoothness and belonging to the Hardy space $H_{q,\rho}$ are given.

Keywords: best simultaneous approximation, Hardy space, upper bound, modulus of smoothness, majorant.

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