Trudy Instituta Matematiki i Mekhaniki UrO RAN

Vol. 29 No. 4 2023

MSC: 05E30, 05C50

DOI: 10.21538/0134-4889-2023-29-4-279-282

ON GRAPHS IN WHICH THE NEIGHBORHOODS OF VERTICES ARE EDGE-REGULAR GRAPHS WITHOUT 3-CLAWS

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The triangle-free Krein graph $\mathrm{Kre}(r)$ is strongly regular with parameters $((r^2+3r)^2, r^3+3r^2+r, 0, r^2+r)$. The existence of such graphs is known only for $r=1$ (the complement of the Clebsch graph) and $r=2$ (the Higman–Sims graph). A. L. Gavrilyuk and A. A. Makhnev proved that the graph $\mathrm{Kre}(3)$ does not exist. Later Makhnev proved that the graph $\mathrm{Kre}(4)$ does not exist. The graph $\mathrm{Kre}(r)$ is the only strongly regular triangle-free graph in which the antineighborhood of a vertex $\mathrm{Kre}(r)'$ is strongly regular. The graph $\mathrm{Kre}(r)'$ has parameters $((r^2+2r-1)(r^2+3r+1), r^3+2r^2, 0, r^2)$. This work clarifies Makhnev's result on graphs in which the neighborhoods of vertices are strongly regular graphs without 3-cocliques. As a consequence, it is proved that the graph $\mathrm{Kre}(r)$ exists if and only if the graph $\mathrm{Kre}(r)'$ exists and is the complement of the block graph of the quasi-symmetric 2-design.

Keywords: distance-regular graph, strongly regular graph.

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Received August 22, 2023
Revised September 12, 2023
Accepted September 18, 2023

Funding Agency: This work was supported by the National Natural Science Foundation of China (project no. 12171126) and by a grant from the Engineering Modeling and Statistical Computing Laboratory of the Hainan Province.

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Cite this article as: M. Chen, A. A. Makhnev, M. S. Nirova. On graphs in which the neighborhoods of vertices are edge-regular graphs without 3-claws. *Trudy Instituta Matematiki i Mekhaniki UrO RAN*, 2023, vol. 29, no. 4, pp. 279–282.