

MSC: 05E18, 20B25

DOI: 10.21538/0134-4889-2023-29-4-274-278

A GRAPH WITH A LOCALLY PROJECTIVE VERTEX-TRANSITIVE GROUP OF AUTOMORPHISMS $\text{Aut}(Fi_{22})$ WHICH HAS A NONTRIVIAL STABILIZER OF A BALL OF RADIUS 2**V. I. Trofimov**

Earlier, to confirm that one of the possibilities for the structure of vertex stabilizers of graphs with projective suborbits is realizable, we announced the existence of a connected graph Γ admitting a group of automorphisms G which is isomorphic to $\text{Aut}(Fi_{22})$ and has the following properties. First, the group G acts transitively on the set of vertices of Γ , but intransitively on the set of 3-arcs of Γ . Second, the stabilizer in G of a vertex of Γ induces on the neighborhood of this vertex a group $PSL_3(3)$ in its natural doubly transitive action. Third, the pointwise stabilizer in G of a ball of radius 2 in Γ is nontrivial. In this paper, we construct such a graph Γ with $G = \text{Aut}(\Gamma)$.

Keywords: graph, transitive locally projective group of automorphisms, Fischer group Fi_{22} .

REFERENCES

1. Trofimov V.I. Vertex stabilizers of locally projective groups of automorphisms of graphs: a summary. In: *Groups, Combinatorics and Geometry, Durham 2001*, NJ etc.: World Sci. Publ., 2003. P. 313–326.
2. Trofimov V.I. Graphs with projective suborbits. Cases of small characteristics. I, II. *Russian Academy of Sciences Izvestiya Mathematics*, 1995, vol. 45, no. 2, pp. 353–398. doi: 10.1070/IM1995v045n02ABEH001645; 1995, vol. 45, no. 3, pp. 559–576. doi: 10.1070/IM1995v045n03ABEH001672
3. Trofimov V.I. Vertex stabilizers of graphs with projective suborbits. *Dokl. Math.*, 1991, vol. 42, no. 3, pp. 825–828.
4. Trofimov V.I. Graphs with projective suborbits. Exceptional cases of characteristic 2, I. *Russian Academy of Sciences Izvestiya Mathematics*, 1998, vol. 62, no. 6, pp. 1221–1279. doi: 10.1070/IM1998v062n06ABEH000224
5. Conway J.H. [et al.]. *Atlas of finite groups*. Oxford: Clarendon Press, 1995. 252 p.
6. Wilson R.A. On maximal subgroups of the Fischer group Fi_{22} . *Math. Proc. Cambridge Philos. Soc.*, 1984, vol. 95, pp. 197–222. doi: 10.1017/S0305004100061491
7. Kleidman P.B., Wilson R.A. The maximal subgroups of Fi_{22} . *Math. Proc. Cambridge Philos. Soc.*, 1987, vol. 102, pp. 17–23. doi: 10.1017/S0305004100067001
8. Kitazume M., Yoshiara S. The radical subgroups of the Fischer simple groups. *J. Algebra*, 2002, vol. 255, pp. 22–58. doi: 10.1016/S0021-8693(02)00119-9
9. Fischer B. *Finite groups generated by 3-transpositions*. WMI Preprints, Coventry (UK): University of Warwick, 1969.
10. Gardiner A. Arc transitivity in graphs. *Quart. J. Math. Oxford (2)*, 1973, vol. 24, pp. 399–407. doi: 10.1093/qmath/24.1.399

Received September 26, 2023

Revised October 6, 2023

Accepted October 9, 2023

Funding Agency: This work was performed as a part of the research conducted in the Ural Mathematical Center and supported by the Ministry of Education and Science of the Russian Federation (agreement no. 075-02-2023-935).

Vladimir Ivanovich Trofimov, Dr. Phys.-Math. Sci., Krasovskii Institute of Mathematics and Mechanics of the Ural Branch of the Russian Academy of Sciences, Yekaterinburg, 620108 Russia; Prof., Ural Federal University, Yekaterinburg, 620083 Russia, e-mail: trofimov@imm.uran.ru.

Cite this article as: V.I. Trofimov. A graph with a locally projective vertex-transitive group of automorphisms $\text{Aut}(Fi_{22})$ which has a nontrivial stabilizer of a ball of radius 2. *Trudy Instituta Matematiki i Mekhaniki UrO RAN*, 2023, vol. 29, no. 4, pp. 274–278.