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SHARP CARLSON TYPE INEQUALITIES WITH MANY WEIGHTS

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The paper is concerned with sharp Carlson type inequalities of the form

$$\|w(\cdot)x(\cdot)\|_{L_q(T)} \leq K \|w_0(\cdot)x(\cdot)\|_{L_p(T)}^\gamma \max_{1 \leq j \leq n} \|w_j(\cdot)x(\cdot)\|_{L_r(T)}^{1-\gamma},$$

where T is a cone in \mathbb{R}^d and the weight functions $w_j(\cdot)$, $j = 1, \dots, n$, are homogeneous with some symmetry property.

Keywords: Carlson type inequalities, sharp constants.

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