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## OPTIMAL INTERPOLATION ON AN INTERVAL WITH THE SMALLEST MEAN-SQUARE NORM OF THE $R$ TH DERIVATIVE

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An exact solution is found to the problem of interpolation on a finite interval  $[a, b]$  with the smallest  $L_2$ -norm of the  $r$ th-order derivative ( $r \geq 2$ ) by functions  $f: [a, b] \rightarrow \mathbb{R}$  with absolutely continuous  $(r - 1)$ th-order derivatives for finite collections of data from the unit ball of the space  $l_2^N$ . Interpolation is performed at nodes of an arbitrary grid  $\Delta_N: a = x_1 < x_2 < \dots < x_N = b$ . The smallest value of the  $L_2$ -norm on the class of interpolated data is expressed in terms of the largest eigenvalue of a certain square matrix and its determinant. The paper improves the classical results of spline theory related to the minimum norm property, which were originally obtained by J. Holladay and then developed by J. Ahlberg, E. Nilson, and J. Walsh, as well as by V. N. Malozemov and A. B. Pevnyi.

Keywords: interpolation, natural splines, matrix eigenvalue.

### REFERENCES

1. Holladay J. A smoothest curve approximation. *Math. Tables Aids Comput.*, 1957, vol. 11, pp. 233–243.
2. Ahlberg J.H., Nilson E.N., Walsh J.L. *The theory of splines and their applications*. NY, London: Acad. Press, 1967. Translated to Russian under the title *Teoriya splainov i ee prilozheniya*, Moscow: Mir Publ., 1972, 316 p.
3. Malozyomov V.N., Pevny A.B. *Polynomial'nye splainy* [Polynomial splines]. Leningrad, 1986, 120 p.
4. Tikhomirov V.M., Boyanov B.D. Some convex problems of approximation theory. *Serdica Bulgariacae mathematicae publicationes*, 1979, vol. 5, pp. 83–96 (in Russian).
5. Subbotin Yu.N. Functional interpolation in the mean with the smallest  $n$ -th derivative. *Tr. Math. Inst. Steklov*, 1967, vol. 88, pp. 30–60 (in Russian).
6. Subbotin Yu.N., Novikov S.I., Shevaldin V.T. Extremal function interpolation and splines. *Tr. Inst. Mat. Mekh. UrO RAN*, 2018, vol. 24, no. 3, pp. 200–225 (in Russian). doi: 10.21538/0134-4889-2018-24-3-200-225
7. Subbotin Yu.N., Shevaldin V.T. Extremal functional  $L_p$  interpolation on an arbitrary mesh on the real axis. *Sb. Math.*, 2022, vol. 213, no. 4, pp. 556–577. doi: 10.1070/SM9628
8. Novikov S.I. Periodic interpolation with minimal norm of  $m$ -th derivative. *Sib. Zhurn. Vychis. Math.*, 2006, vol. 9, no. 2, pp. 165–172 (in Russian).
9. Schoenberg I.J. On the best approximation of linear operators. *Indagationes Mathem.* 1964. Vol. 26, no 2. P. 155–163.
10. Jerome J.W., Schumaker L.L. A note on obtaining natural spline functions by the abstract approach of Atteia and Laurent. *SIAM J. Numer. Anal.*, 1968, vol. 5, pp. 657–663.
11. Prudnikov A.P., Brychkov A.Yu., Marichev O.L. *Integrals and series. Vol. 1: Elementary functions*. Boca Raton: CRC Press, 1998, 798 p. Original Russian text published in Prudnikov A.P., Brychkov Yu.A., Marichev O.I. *Integraly i ryady. T. 1. Elementarnye funktsii*. Moscow: Nauka Publ., 1981, 800 p.
12. Holmes R. *Geometric functional analysis and its applications*. NY ect.: Springer Verlag, 1975, 246 p.
13. Fichtenholz G.M. Fikhtenholtz G.M. *Kurs differentsial'nogo i integral'nogo ischisleniya* [Course of differential and integral calculus]. Vol. 1. Moscow: Nauka Publ., 1970, 608 p.
14. Faddeev D.K., Faddeeva V.N. *Vychislitel'nye metody lineinoi algebrы* [Numerical methods in linear algebra]. St. Petersburg: "Lan'" Publ., 2009, 736 p.

15. Parodi M. *A localisation des valeurs caracteristiques des matrices et ses applications*. Paris: Gauthier—Villars, 1959. Translated to Russian under the title *Lokalizatsiya kharakteristicheskikh chisel matrits i ee primeneniya*, Moscow: Iz-vo Inostr. Liter. Publ., 1960, 170 p.
16. Tarazaga P. Eigenvalue estimates for symmetric matrices. *Linear Algebra and its Appl.*, 1990, vol. 135, no. 1, pp. 171–179. doi: 10.1016/0024-3795(90)90120-2

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