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ON THE CONNECTION BETWEEN CLASSES OF FUNCTIONS OF BOUNDED VARIATION AND CLASSES OF FUNCTIONS WITH FRACTAL GRAPH

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For a real-valued function f continuous on a closed interval, the modulus of fractality $\nu(f,\varepsilon)$ is defined for every $\varepsilon > 0$ as the minimum number of squares with sides of length ε parallel to the coordinate axes that can cover the graph of f. For a nonincreasing function $\mu : (0,+\infty) \to (0,+\infty)$, we consider the class F^{μ} of functions continuous on a closed interval and such that $\nu(f,\varepsilon) = O(\mu(\varepsilon))$. The relationship between the classes F^{μ_1} and F^{μ_2} is described for various μ_1 and μ_2. A connection is established between the classes F^{μ} and the classes of continuous functions of bounded variation $BV_{\Phi}[a,b] \cap C[a,b]$ for arbitrary convex functions Φ. Namely, there is an inclusion

$$BV_{\Phi}[a,b] \cap C[a,b] \subset F^{\frac{\Phi^{-1}(\varepsilon)}{\varepsilon^2}}.$$

A counterexample is constructed showing that this inclusion cannot be improved. It is further shown that the equality of the classes F^{μ} and $BV_{\Phi}[a,b] \cap C[a,b]$ occurs only in the case

$$BV[a,b] \cap C[a,b] = F^{1/\varepsilon},$$

where $BV[a,b]$ are functions of classical bounded variation. For other cases, a counterexample is constructed showing that if $\mu(\varepsilon)$ grows faster than $\dfrac{1}{\varepsilon}$ as $\varepsilon \to +0$, then the class F^{μ} is not a subclass of any of the classes $BV_{\Phi}[a,b]$.

Keywords: fractal dimension, bounded variation.

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