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ON THE KEGEL–WIELANDT σ -PROBLEM

S. F. Kamornikov, V. N. Tyutyaynov

For an arbitrary partition σ of the set \mathbb{P} of all primes, a sufficient condition for the σ -subnormality of a subgroup in a finite group is given. It is proved that the Kegel–Wielandt σ -problem has a positive solution in the class of all finite groups all of whose nonabelian composition factors are alternating groups, sporadic groups, or Lie groups of rank 1.

Keywords: finite group, σ -subnormal subgroup, Kegel–Wielandt σ -problem, Hall subgroup, complete Hall set.

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Sergei Fedorovich Kamornikov, Dr. Phys.-Math. Sci., Prof., F. Skorina Gomel State University, Gomel, 246028 Republic of Belarus, e-mail: sfkamornikov@mail.ru.

Valentin Nikolayevich Tyutyaynov, Dr. Phys.-Math. Sci., Prof., Gomel Branch of International University “MITSO”, Gomel, 246029 Republic of Belarus, e-mail: vtutanov@gmail.com.

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