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ON PRODUCTS OF π -SOLVABLE FINITE GROUPS

L. S. Kazarin

In this paper, we study finite groups having a triple factorization $G = AB = AC = BC$, where the factors A , B , and C are π -solvable subgroups of the group G for some set π of primes. This problem seems to have been first formulated by A. F. Vasil'ev and A. K. Furs in 2021 at the conference dedicated to the 90th anniversary of the birth of A. I. Starostin.

Keywords: finite group, subgroup, character, representation, factorization.

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Lev Sergeevich Kazarin, Dr. Phys.-Math. Sci., Prof., Yaroslavl P. Demidov State University, Yaroslavl, 150001 Russia, e-mail: lsk46@mail.ru .

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